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# Analysis of Shared Parking Game Model under Dynamic Parking Pricing

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**Abstract:** In recent years, the problem of "parking difficulty" has led to a large number of illegal parking incidents. The shared parking mode has been considered as an effective way to alleviate the conflict between parking supply and demand and urban traffic pressure. To address the issue of illegal parking and promote the development of shared parking, this paper constructs an evolutionary game model with shared platforms and motor vehicle drivers as the main entities. The study investigates the evolutionary stability strategy of the model, conducts sensitivity analysis on model parameters, and further analyzes the impact of highly sensitive parameters on the evolutionary paths of both players in the game. Finally, numerical simulations are performed on dynamic parking pricing standard. The research findings demonstrate that the sensitivity of discounts received by drivers from shared platforms and the additional revenue gained by the shared platform is higher than that of other parameters. Moderately increasing the penalty for illegal parking and the additional revenue of the shared platform can encourage drivers to choose legal parking and promote the development of shared parking. Under given parameterized and periodic parking pricing standard, finally, according to the particle swarm optimization algorithm, a set of relatively optimal parameter values is derived to enable the model to evolve rapidly into a stable state where drivers choose to park legally and the shared platform selects surrounding parking lots. It can effectively reduce the frequency of illegal parking.

**Keywords:** Evolutionary Game; Dynamic Parking Pricing; Shared Parking; Sensitivity Analysis

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## 1. Introduction

With the continuous improvement of urbanization and residents' living standard, the number of motor vehicles in China has increased rapidly, while the supply of parking spaces remains relatively insufficient, leading to an increase in parking demand and a large number of illegal parking incidents. Shoup's [1] research pointed out that if it took three minutes to find a parking space per parking activity, it would add about 1,825 kilometers of cruising distance per car per year. Not only that, car cruise parking was responsible for about 30% of traffic congestion in the city centers of the 11 cities surveyed, with an average cruise time of 8.1 minutes per vehicle [1]. In addition, in 2017, the demand for overnight parking in the central urban area of Shanghai was 1.33 million vehicles, while the built parking spaces for residential purposes were only 640,000, with a gap of 52% [2]. These data show that an insufficient supply of parking spaces can cause some drivers to spend more time looking for parking spaces, while others will choose to park illegally. Therefore, if this problem can be solved, it will not only effectively alleviate the driver's parking pressure, but also reduce the pollution caused by vehicles to the environment.

However, due to the increasing number of motor vehicles and the decreasing land area, it is not feasible to continue to build new parking facilities to solve the parking problem [3]. Because the problem of parking difficulties is not only caused by the shortage of parking spaces, parking space information sharing and resource allocation are not in place, which will also lead to parking difficulties [4]. For example, there were 485,000 private parking spaces in Hong Kong, accounting for nearly 70% of the total of parking spaces; Beijing's residential parking resources account for 58.1% of all parking resources, and nearly 800,000 residential parking spaces have been left unused during working hours [5]. In addition, in 2019, the average vacancy rate of parking lots in China was 51.3%, and the nighttime utilization rate of parking spaces in Nanjing's public institutions and government offices was only 40% to 50% [2]. These data indicate that the existing urban parking space turnover rate is low and a large number of parking spaces are wasted and left unused. In real life, if idle private and residential parking spaces can be effectively utilized, it will be able to alleviate the parking problem.

In recent years, as sharing has become more popular, the shared parking model has gradually emerged. As an effective measure to rationally utilize idle parking spaces in different areas and alleviate the relative shortage of parking spaces, shared parking has been widely used to solve the problem of illegal parking. The sharing platform plays a crucial role in sharing parking by collecting parking space information from parking space owners and distributing parking space information to users. Shared platforms integrate real-time information about parking spaces in urban areas and popular tourist attractions, enabling users to quickly search for destinations and nearby parking lots through authorized apps or mini-programs provided by various sharing platforms [6]. According to factors such as the distance between the parking lot and the destination, the availability of parking spaces, parking costs, and traffic conditions on the way to the parking lot, the system recommends parking spaces and routes with the greatest benefits to the driver and provides accurate navigation guidance to help the driver find a parking space, solve the problem of "parking difficulty", and encourage the driver to drive to the destination[6]. Xu et al [7] are one of the earlier groups to use shared private parking spaces to benefit society, and their paper opens the door to using shared parking to solve parking difficulties.

So far, some scholars have focused on studying shared parking from the perspective of parking fees. Hao et al [8] established a parking behavior choice model and proposed a dynamic balance adjustment method of floating charges for shared parking, which found that this method not only ensured the drivers' demand for parking, but also fully exploited the potential of shared parking in solving the parking difficulty problem. Ayala et al [9] presented a vs-pricing scheme, which found that this scheme resulted in a 23% increase in the total driving distance. Qian et al [10] modeled the relationship between parking pricing and perception, put forward the optimal parking pricing strategy, and found a pricing strategy that could cope with different demand levels and was generally better than the deterministic pricing scheme. The research of these scholars helps to attract more residents who own parking spaces and drivers to integrate into shared parking mode while minimizing the total social cost through their research on parking pricing. In this paper, dynamic parking charge is introduced into the model in the form of function to adapt to different periods of parking demand and attract drivers to park legally.

In addition, there are some scholars who also study the game between private parking space owners, sharing platforms, drivers, and other groups according to game theory. Duan [11] conducted in-depth research on the residential area's shared parking game model and constructed a dynamic game model with the shared platform and drivers as the main body, which obtained the shared parking scheme as the Nash equilibrium solution for the game. Ma [12] considered the needs of both parking space providers and the shared parking management platform, introducing the Stackelberg game to describe their relationship and studying the allocation of shared parking spaces. It was found that the optimized shared parking space allocation model can effectively reduce the rejection rate and manage parking conflicts among users. Based on the bargaining theory, Hu [13] established a parking fee game model and solved the model by using the inverse classification method. Meanwhile, by analyzing the feasibility of parking space sharing, people can improve their understanding of the parking space sharing theory and its potential wide application. Li [14] established a three-party bargaining revenue game model involving the sharing platform, property management, and parking space owners, and solved it to obtain the optimal revenue proportions for each party. It further combined the analysis of the number of shared parking spaces and obtained the relationship between the characteristics of the owners of parking spaces and the income of the shared platform and the property. Said et al [15] proposed a solution for a green intelligent parking system based on the Internet of Things (IoT) and established a reservation system model in the solution according to game theory. It ultimately addressed issues such as parking fees, parking lot distance from the driver's destination, walking distance, and parking time when drivers were looking for available parking spaces. Du [16] established a stochastic Poisson game to simulate the competition between in-route parking vehicles in multiple parking facilities, and introduced a decentralized and coordinated online parking mechanism, finding that DCPM performs better than three greedy strategies, following the nearest first, the cheapest first, and the least cruising first strategy respectively.

In terms of research on shared parking, existing articles based on game theory involve a large

number of groups and have made significant contributions to the development of shared parking and the improvement of social order. However, few have studied the game between drivers and shared platforms. In order to solve the problem of parking difficulty, the first step is to achieve resource sharing, and the second step is to ensure that drivers park in a legal manner. Therefore, when the shared parking mode is used to solve the problem of illegal parking, it is necessary to study how the shared platform can make motor vehicle drivers park in a legal manner. In other words, it is necessary to study the game between motor vehicle drivers and shared platforms and obtain the conditions that motor vehicle drivers need to meet for legal parking through the game. The sharing platform releases real-time parking space information based on driver needs, and drivers make preliminary decisions based on the received parking space information. As the sharing platform updates parking space information, drivers will adjust their decisions at any time. Therefore, the game between the two is dynamic. **Figure 1** shows the relationship between the drivers and the shared platform. In the process of a long-term game, drivers cannot know exactly their benefits at every moment, and the sharing platform influenced by many factors cannot provide the optimal strategy, so they cannot make decisions directly based on perfect rationality. In fact, compared with the traditional theory, evolutionary game theory can be applied to a game involving participants with bounded rationality and overcome the problem of dynamic game process [17]. Evolutionary game theory is a game theory based on the assumption of the "bounded rationality" of the participants. Participants with bounded rationality may not be able to choose the optimal strategy at the beginning, so they need to constantly learn and adjust the strategy, and finally reach a stable state where all participants choose the optimal strategy [18]. Evolutionary game theory further develops and improves upon classical game theory, and it is widely applied in various fields such as transportation, economics, architecture, and ecology [19-24]. In recent years, with the rise of evolutionary game theory, some scholars began to study the shared parking mode according to it. Jia et al [25] constructed an evolutionary game model for shared parking between parking space owners and drivers, focusing on the influence of different strategy selection ratios and parameters under government encouragement measures on the evolution of the shared parking game. Wang [26] constructed a tripartite evolutionary game model and took the government, the company, and the parking space owners as participants, finding that changes in government revenue and costs significantly affected the government's "incentive" strategy choice, while the proportion of companies choosing the "create shared parking platform" strategy was influenced by the changes in their operating costs and potential benefits.

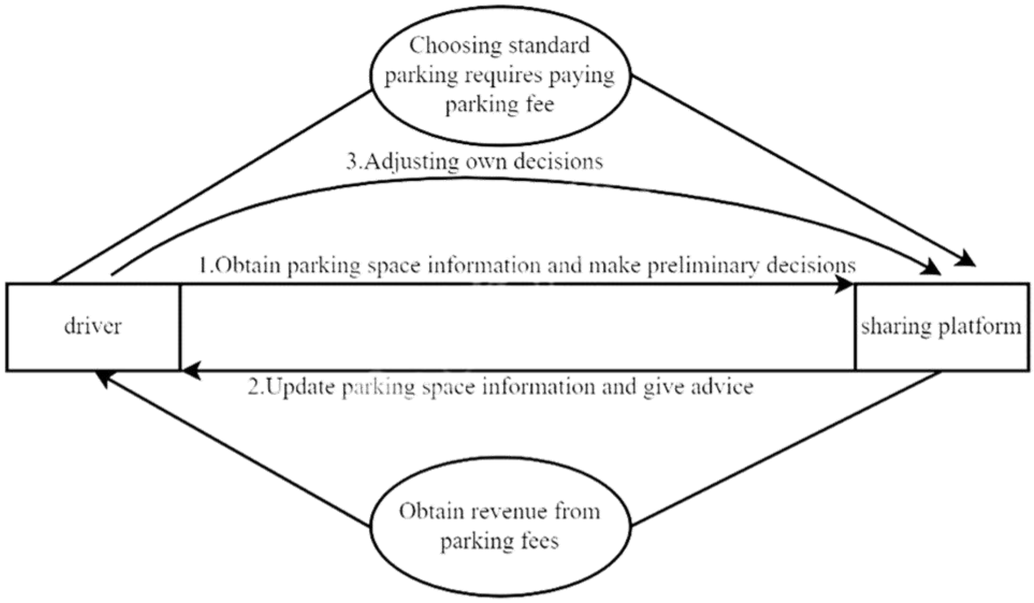



**Figure 1.** The relationship between the drivers and the shared platform

However, there are still few scholars studying shared parking with evolutionary game theory and no scholars have studied the game between drivers and shared platforms based on evolutionary game. This paper aims to use evolutionary game theory to simulate the change of decision making between drivers and sharing platforms over time, analyze the conditions that drivers who choose legal parking should meet when the game is stable, and conduct numerical simulation to finally alleviate the problem of illegal parking caused by difficult parking. This paper makes the following contributions. First, based on evolutionary game theory, this paper constructs an asymmetric evolutionary game model between motor vehicle drivers and sharing platforms and introduces a dynamic parking charging standard in the form of function. It explores the model's evolutionary stable strategies. Second, through numerical simulation, this paper discusses the influence of different values of the key parameters of the model on the decision of both parties and the influence of different values of the parameters in the parking fee standard on the decision of both parties, and further obtains the optimal parking fee standard based on particle swarm optimization algorithm. This provides a theoretical reference for the government to formulate relevant policies and enterprises to develop sharing platforms.

The specific content of this paper is organized as follows: Section 2 introduces the research problem and basic assumptions, establishing the game model between the sharing platform and drivers; Section 3 studies the evolutionary stability of equilibrium points to find the evolutionary stable strategies; Section 4 conducts sensitivity analysis on the parameters of the model; In Section 5, through numerical simulation, the influence of different parameter values on the evolution path of two players' decisions and the influence of dynamic parking charging standard on its evolution path are explored, and the local optimal charging standard is obtained in accordance with the particle swarm optimization algorithm. The conclusions and suggestions are given in Section 6.

## 2. Evolutionary game model of motor vehicle drivers and sharing platforms

Urban residential and commercial areas involve the participation of two main entities, namely, motor vehicle drivers and sharing platforms, in the process of shared parking management. They have mutual influences and constraints on each other's decisions. In order to more realistically reflect the dynamic process of mutual influence between these two entities in the management process, this paper establishes an evolutionary game model between these two participants. The model is subjected to evolutionary stability analysis, and some recommendations are provided.

### *2.1. Model assumptions and parameter descriptions*

To facilitate the description of the problem and the establishment of the model, the following reasonable assumptions are made:

(1) The assumption is that both the motor vehicle drivers and the sharing platforms have bounded rationality. As parking information is updated, their strategy is not necessarily optimal at first. With the passage of time, they can reach a stable state in the evolutionary game model of shared parking through self-learning.

(2) The assumption is that motor vehicle drivers are not sure whether there is a parking space available before arriving at the destination. In order to reduce travel costs, they will use shared platforms to get optimal parking spaces and go to the parking lot recommended by the shared platform.

(3) The strategies of motor vehicle drivers are legal parking and illegal parking, while the strategies of the sharing platform are destination parking lots and surrounding parking lots.

(4) Assuming that drivers make decisions from their interests based on factors such as the level of parking costs, the convenience of parking, and walking distance after parking, with probabilities of legal parking and illegal parking being $x$ and $1-x$ respectively. Additionally, the sharing platform makes decisions based on factors such as the ratio of remaining parking spaces in current parking lots, gains and losses of motor vehicle drivers, and its gains and losses. The probabilities of the sharing platform choosing surrounding parking lots and destination parking lots are $y$ and $1-y$ respectively, where $x, y \in [0,1]$ and $x, y$ are both functions of time.

In this evolutionary model, $F$ and $U$ respectively represent the fines that the drivers need to pay for illegal parking and the loss of vehicle damage caused by illegal parking (such as scratches). $P_1$ and $P_2$ respectively represent the probability of the drivers being caught parking illegally and the probability of the vehicle being injured due to illegal parking. Drivers will be punished for illegal parking, while drivers who park legally will receive hidden benefits. $C_1$ (related to fines for violations and vehicle damage) represents the hidden benefits obtained by drivers. It's called safety benefits. $S$ represents the distance from the destination after the drivers get off the vehicle, and $\alpha$ represents the coefficient by which distance is converted into expenditure. $D$ represents the discount received by drivers when paying parking fees on the shared platform. $g(t)$ and $h(t)$ respectively represent the parking fees paid by drivers in surrounding parking lots and destination parking lots. $K_1$ and $K_2$ respectively represent the parking fee standard for the destination parking lot and surrounding parking lots. $\triangle t$ represents the average parking time of the drivers in the parking lot, and $P_3$ represents that if the drivers park legally, the shared platform will receive a certain proportion of the parking fee. $C_2$ represents the basic operating expenses of the shared platform, while $C_3$ and $E$ respectively represent the additional expenses and income generated by the shared platform due to sharing.

**Table 1** shows the required parameters for the model.

**Table 1.** The meaning of each parameter

| Classification | Parameter | Meanings |
|---|---|---|
| | $F$ | Violation fines |
| Driver | $P_1$ | Probability of punishment for violations |
| | $U$ | The damage cost incurred by the illegally parked vehicle |

| | | |
|---|---|---|
| | $P_2$ | The probability of vehicle damage |
| | $C_1$ | The safety benefits gained by drivers |
| | $\alpha$ | The walking distance conversion index |
| | $S$ | The increased walking distance due to shared parking |
| | $D$ | The discount that drivers receive from the sharing platform |
| | $g(t)$ | The parking fees paid by drivers at surrounding parking lots |
| | $h(t)$ | The parking fees paid by drivers at the destination parking lot |
| Sharing Platform | $K_1$ | The parking fee rates at the destination parking lot |
| | $K_2$ | The parking fee rates at the surrounding parking lots |
| | $\vartriangle t$ | Average parking time |
| | $P_3$ | The coefficient of profit obtained by the sharing platform from parking fees |
| | $C_2$ | The operational base cost of the sharing platform |
| | $C_3$ | The additional operational cost incurred by the sharing platform due to the sharing |
| | $E$ | The additional revenue generated by the sharing platform due to sharing |

To accurately represent the parking fees over a certain period, this paper introduces Riemann integration. The Riemann integral can be represented as $\lim_{\|T\|\to 0}\sum_{i=1}^{n} f(\xi_i)\vartriangle t_i = \int_a^b f(t)dt$. For the model in this paper, it can be understood as the sum of parking fees for each small interval of time within the duration $\vartriangle t = b - a$. We will use this integral to represent the parking charges.

In the context of the classical evolutionary game model, this paper introduces dynamic parking fee criteria to achieve dynamic parking charges. The dynamic parking fee criteria for the destination parking lot and the surrounding parking lots are $K_1, K_2$, the corresponding dynamic parking fees are $h(t) = \int_t^{t+\vartriangle t} K_1(t_1)dt_1$, $g(t) = \int_t^{t+\vartriangle t} K_2(t_1)dt_1$.

Based on the above assumptions, the payoff matrix can be constructed for the shared parking game model, as shown in **Table 2**.

**Table 2.** Payoff matrix

| sharing platform | driver | |
|---|---|---|
| | legal parking(x) | illegal parking(1-x) |
| surrounding parking lot (y) | $P_3 g(t) + E - C_2 - C_3$, $C_1 - Dg(t) - \alpha S$ | $E - C_2 - C_3$, $-P_1F - P_2U - \alpha S$ |
| destination parking lot (1-y) | $P_3 h(t) - C_2$, $C_1 - Dh(t)$ | $-C_2$, $-P_1F - P_2U$ |

### *2.2. Evolutionary game model construction*

Based on the payoff matrix, we construct the replicator dynamics equation between the two participants in the shared parking scenario as follows:

#### 2.2.1. The replicator dynamics equation for the shared platform's choice of the surrounding parking lot strategy

The expected revenue for the shared platform in selecting the surrounding parking lot,

$$W_1 = x(P_3 g(t) + E - C_2 - C_3) + (1-x)(E - C_2 - C_3) = = xP_3 g(t) + E - C_2 - C_3. \tag{1}$$

Similarly, we can obtain the expected revenue for the shared platform in selecting the destination parking lot,

$$W_2 = x(P_3 h(t) - C_2) + (1-x)(-C_2). \tag{2}$$

Based on equations (1) and (2), we can deduce the average revenue for the shared platform,

$$W = yW_1 + (1-y)W_2 = y(xP_3 g(t) + E - C_3) + (1-y)xP_3 h(t) - C_2. \tag{3}$$

In conclusion, the replicator dynamics equation for the shared platform's choice of the surrounding parking lot is derived as follows,

$$F(y) = y(W_1 - W) = y(1-y)[x(P_3 g(t) - P_3 h(t)) + E - C_3]. \tag{4}$$

**2.2.2. The replicator dynamics equation for motor vehicle drivers' choice of legal parking strategy**

The expected benefits for drivers choosing legal parking,

$$V_1 = y(C_1 - Dg(t) - \alpha S) + (1-y)(C_1 - Dh(t)). \tag{5}$$

The expected benefits for drivers choosing illegal parking,

$$V_2 = y(-P_1 F - P_2 U - \alpha S) + (1-y)(-P_1 F - P_2 U) = -y\alpha S - P_1 F - P_2 U. \tag{6}$$

Based on equations (5) and (6), we can deduce the average revenue for the driver,

$$V = xV_1 + (1-x)V_2 = x[C_1 - D(yg(t) + (1-y)h(t))] + (1-x)(-P_1 F - P_2 U) - y\alpha S. \tag{7}$$

In conclusion, the replicator dynamics equation for motor vehicle drivers' choice of legal parking is as follows,

$$G(x) = x(V_1 - V) = x(1-x)[y(Dh(t) - Dg(t)) + C_1 + P_1 F + P_2 U - Dh(t)]. \tag{8}$$

Combining equations (4) and (8), we obtain the evolutionary game model for shared parking,

$$\begin{cases} \dfrac{dx}{dt} = x(1-x)(\Phi_1 y - \Psi_1), \\ \dfrac{dy}{dt} = y(1-y)(\Phi_2 x - \Psi_2), \end{cases} \tag{9}$$

where,

$$\begin{cases} \Phi_1 = D(h(t) - g(t)), \\ \Psi_1 = Dh(t) - C_1 - P_1 F - P_2 U, \\ \Phi_2 = P_3(g(t) - h(t)), \\ \Psi_2 = C_3 - E. \end{cases} \tag{10}$$

## 3. Stability analysis of the evolutionary dynamics between motor vehicle drivers and the shared platform

The two most important concepts in evolutionary game theory are Evolutionary Stability Strategy (ESS) and replicator dynamics [27]. ESS describes that if the majority of individuals in a group choose this strategy, then a small variation group consisting of other strategies cannot invade this group [28]. From the system point of view, the ESS of the game is a stable equilibrium of the system [28]. To determine whether the equilibrium points of the model are ESS, in this section, we will investigate the stability of the evolutionary equilibrium points to study the ESS of the shared parking game model proposed in this paper.

In the previous section, by setting $\frac{dx}{dt}=0$ and $\frac{dy}{dt}=0$ in equation (9), we obtain five equilibrium points for the shared parking game model: $Z_0=(0,0)$, $Z_1=(0,1)$, $Z_2=(1,0)$, $Z_3=(1,1)$, $Z_*=(\frac{\Psi_2}{\Phi_2},\frac{\Psi_1}{\Phi_1})$.

The equilibrium point $Z_0$ indicates that in the long-term process of the game, the system will evolve to a state where drivers engage in illegal parking and the shared platform chooses the destination parking lot, which implies that illegal parking will occur frequently, posing significant harm to social order. The equilibrium point $Z_1$ represents the system will evolve to a state where drivers engage in legal parking and the shared platform chooses the destination parking lot. The equilibrium point $Z_2$ indicates that the system will evolve to a state where drivers engage in illegal parking, and the shared platform chooses the surrounding parking lots. The equilibrium point $Z_3$ indicates that in the long-term process of the game, the system will evolve to a state where drivers engage in legal parking, and the shared platform chooses the surrounding parking lots. This implies that the phenomenon of illegal parking will not occur, and traffic orders will improve. The equilibrium point $Z_*$ signifies that in the long-term process of the game, there will be a coexistence of drivers tending to park either legally or illegally, and the shared platform will not exclusively choose either the destination parking lot or the surrounding parking lots.

In evolutionary game theory, a point is considered an evolutionarily stable point when its Jacobi determinant is greater than 0 and its trace is less than 0. On the other hand, a point is deemed unstable when both its Jacobi determinant and trace are greater than 0 [18].

By taking the partial derivatives of F(y) and G(x) concerning their corresponding variables, we obtain the Jacobi matrix,

$$J=\begin{bmatrix}(1-2x)(\Phi_1 y-\Psi_1) & \Phi_1 x(1-x)\\ \Phi_2 y(1-y) & (1-2y)(\Phi_2 x-\Psi_2)\end{bmatrix}. \tag{11}$$

Furthermore, we can obtain their determinant and trace, which are respectively,

$$\left|J\right|=(1-2x)(1-2y)(\Phi_2 x-\Psi_2)(\Phi_1 y-\Psi_1)-xy(1-x)(1-y)\Phi_1\Phi_2, \tag{12}$$

$$\begin{cases}\Phi_1=D(h(t)-g(t)),\\ \Psi_1=Dh(t)-C_1-P_1F-P_2U,\\ \Phi_2=P_3(g(t)-h(t)),\\ \Psi_2=C_3-E.\end{cases} \tag{13}$$

**Theorem 3.1.** If $\Psi_1>0$ and $\Psi_2>0$, then $Z_0$ is an ESS.

Proof. Substituting the equilibrium point $Z_0$ into the formulas (12) and (13), we obtain,

$$\left|J\right|=\Psi_1\Psi_2,\quad trJ=-\Psi_1-\Psi_2. \tag{14}$$

If $\Psi_1>0$ and $\Psi_2>0$, then $\left|J\right|>0$ and $trJ<0$, so $Z_0$ is an ESS.

When drivers are considering adopting the legal parking strategy, if the parking fee they need to pay at the destination parking lot exceeds the sum of their expected safe benefits from legal parking and the fixed loss from illegal parking, then they will resort to illegal parking. On the other hand, when the shared platform incurs additional expenses due to sharing that outweigh the generated income, it will tend to choose the destination parking lot strategy. In the long game, even if a minority of drivers and the shared platform initially choose the legal parking and surrounding parking lots strategies, eventually, they will switch to the other strategy. At that point, social order will be severely disrupted.

**Theorem 3.2.** (1) If $\Psi_2 < 0$ and $\Phi_1 < \Psi_1$, then $Z_1$ is an ESS.

(2) If $\Psi_1 < 0$ and $\Phi_2 < \Psi_2$, then $Z_2$ is an ESS.

Proof. Substituting the equilibrium points $Z_1$ and $Z_2$ into the formulas (12) and (13), we obtain,

$$\left|J\right|_{Z_1} = \Psi_2(\Phi_1 - \Psi_1), \quad trJ_{Z_1} = \Phi_1 - \Psi_1 + \Psi_2, \tag{15}$$

$$\left|J\right|_{Z_2} = \Psi_1(\Phi_2 - \Psi_2), \quad trJ_{Z_2} = \Psi_1 + \Phi_2 - \Psi_2. \tag{16}$$

If $\Psi_2 < 0$ and $\Phi_1 < \Psi_1$, then $\left|J\right|$ >0 and $trJ$ <0, so $Z_1$ is an ESS; If $\Psi_1 < 0$ and $\Phi_2 < \Psi_2$, then $\left|J\right|$ >0 and $trJ$ <0, so $Z_2$ is an ESS.

Since the descriptions of the evolutionary stable states for equilibrium points $Z_1$ and $Z_2$ are similar to the point $Z_0$, there is no need to repeat them here.

**Theorem 3.3.** If $\Phi_1 > \Psi_1$ and $\Phi_2 > \Psi_2$, then $Z_3$ is an ESS.

Proof. Substituting the equilibrium point $Z_3$ into the formulas (12) and (13), we obtain,

$$\left|J\right| = (\Phi_1 - \Psi_1)(\Phi_2 - \Psi_2), \quad trJ = -(\Phi_1 - \Psi_1) - (\Phi_2 - \Psi_2). \tag{17}$$

If $\Phi_1 > \Psi_1$ and $\Phi_2 > \Psi_2$, then $\left|J\right|$ >0 and $trJ$ <0, so $Z_3$ is an ESS.

when the parking fee at the surrounding parking lots for drivers is lower than the sum of their expected safe benefits from legal parking and the fixed loss from illegal parking, drivers will adopt the legal parking strategy. Similarly, when the revenue generated from the surrounding parking fees and the additional income from sharing far exceeds the revenue obtained from the destination parking fees for the shared platform, the shared platform will choose the surrounding parking lots strategy. In the long-term process of the game, all drivers will choose legal parking, and the shared platform will opt for the surrounding parking lots. As a result, the system will reach a stable state. At this point, the social order will be highly stable.

**Theorem 3.4.** The equilibrium point $Z_*$ can not be an ESS.

Proof. Substituting the equilibrium point $Z_*$ into the expressions for the formulas (12) and (13), we obtain,

$$\left|J\right| = -\frac{\Psi_1\Psi_2(\Phi_1 - \Psi_1)(\Phi_2 - \Psi_2)}{\Phi_1\Phi_2}, \quad trJ = 0. \tag{18}$$

Due to $trJ = 0$, the point can not be an ESS.

On the basis of the above analysis, we can obtain the stability analysis table for equilibrium points, as shown in **Figure 3**.

**Table 3.** Stability analysis of equilibrium points

| category | $\Psi_1$ | $\Psi_2$ | equilibrium point | \|J\| | tr J | stability |
|---|---|---|---|---|---|---|
| $\Phi_1<\Psi_1,\Phi_2>\Psi_2$ | <0 | <0 | (0,0) | >0 | >0 | instability point |
| | | | (0,1) | >0 | <0 | stable point |
| | | | (1,0) | >0 | >0 | instability point |
| | | | (1,1) | <0 | indeterminacy | saddle point |
| $\Phi_1<\Psi_1,\Phi_2<\Psi_2$ | <0 | >0 | (0,0) | <0 | indeterminacy | saddle point |
| | | | (0,1) | <0 | indeterminacy | saddle point |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | (1,0) | >0 | <0 | stable point |
| | | | (1,1) | >0 | >0 | instability point |
| $\Phi_1 > \Psi_1, \Phi_2 < \Psi_2$ | >0 | >0 | (0,0) | >0 | <0 | stable point |
| | | | (0,1) | >0 | >0 | instability point |
| | | | (1,0) | <0 | indeterminacy | saddle point |
| | | | (1,1) | <0 | indeterminacy | saddle point |
| $\Phi_1 > \Psi_1, \Phi_2 > \Psi_2$ | >0 | <0 | (0,0) | <0 | indeterminacy | saddle point |
| | | | (0,1) | <0 | indeterminacy | saddle point |
| | | | (1,0) | >0 | >0 | instability point |
| | | | (1,1) | >0 | <0 | stable point |

To maintain social order, in real-life scenarios, we would prefer drivers to choose legal parking and the shared platform to select the surrounding parking lots. Therefore, in this shared parking model, we aim for the system to evolve towards point $Z_3$.

According to the above analysis, we can provide the following recommendations:

The transportation management department can lower the cost of legal parking for drivers by modifying relevant regulations and policies, such as reducing parking fees or setting a cap on parking charges. Additionally, the shared platform can encourage drivers to engage in legal parking by periodically offering discounts or coupons to reduce their parking expenses.

The government can encourage research institutions to actively participate in the development and optimization of shared platforms to reduce the operating costs of the shared platform, minimize labor costs and resource wastage, and enhance operational efficiency.

Companies can leverage blockchain technology to create a more secure, reliable, and transparent shared platform. Blockchain can help reduce expenses and transaction costs by eliminating intermediaries and reducing the need for manual record-keeping, thereby enhancing additional revenue generation for the shared platform.

## 4. Analysis of Sensitivity

In real-life scenarios, various factors such as parking fees, penalties for illegal parking, and additional income and expenses for the shared platform can all have an impact on the decisions of drivers and the shared platform. Among these factors, it is essential to pay particular attention to those that have the most significant influence on drivers and the shared platform. By adjusting these critical factors, we can facilitate a quicker evolution of both parties toward the ideal strategy. However, we must also be cautious not to excessively alter these factors. Therefore, for the shared parking model in this paper, conducting a sensitivity analysis of the parameters is necessary, which determines the factors that have the greatest impact on the driver and the shared platform.

The sensitivity equation provided in reference [29] is as follows,

$$\dot{S} = \left[\frac{\partial f}{\partial u}\right]_{\lambda=\lambda_0} S + \left[\frac{\partial f}{\partial \lambda}\right]_{\lambda=\lambda_0}, \tag{19}$$

where $S$ is the sensitivity of the parameter.

Let $Q = E - C_3$ represents the difference between the additional income and expenses generated by sharing for the shared platform, and let $Z = P_1F + P_2U$ denotes the sum of fines for illegal parking and expenses for vehicle damage incurred by drivers. In this model, there are two types of parameters: the cost-type parameters and the proportion-type parameters. We will conduct sensitivity analyses separately for both types of parameters. Let $Q, Z, C_1, h(t), g(t), P_3, D$ are the unknown parameters, and denote them as $\lambda_1 = (Q, Z, C_1, h(t), g(t))$, $\lambda_2 = (P_3, D)$. Let $u = (x, y)$.

The evolutionary game model (9) can be expressed in the following standard form,

$$\begin{cases} \dfrac{dx}{dt} = f_1(\lambda_1, \lambda_2, t, u), \\ \dfrac{dy}{dt} = f_2(\lambda_1, \lambda_2, t, u). \end{cases}$$

The partial derivatives of the nominal equation concerning the solution, denoted as $\dfrac{\partial f}{\partial u}$ and concerning the parameters, denoted as $\dfrac{\partial f}{\partial \lambda_1}, \dfrac{\partial f}{\partial \lambda_2}$, they are given by formulas (20), (21), and (22),

$$\frac{\partial f}{\partial u} = \begin{bmatrix} (1-2x)(\Phi_1 y - \Psi_1) & \Phi_1 x(1-x) \\ \Phi_2 y(1-y) & (1-2y)(\Phi_2 x - \Psi_2) \end{bmatrix}, \tag{20}$$

$$\frac{\partial f}{\partial \lambda_1} = \begin{bmatrix} \dfrac{\partial f_1}{\partial Q} & \dfrac{\partial f_1}{\partial Z} & \dfrac{\partial f_1}{\partial C_1} & \dfrac{\partial f_1}{\partial h(t)} & \dfrac{\partial f_1}{\partial g(t)} \\ \dfrac{\partial f_2}{\partial Q} & \dfrac{\partial f_2}{\partial Z} & \dfrac{\partial f_2}{\partial C_1} & \dfrac{\partial f_2}{\partial h(t)} & \dfrac{\partial f_2}{\partial g(t)} \end{bmatrix}, \tag{21}$$

$$\frac{\partial f}{\partial \lambda_2} = \begin{bmatrix} \dfrac{\partial f_1}{\partial P_3} & \dfrac{\partial f_1}{\partial D} \\ \dfrac{\partial f_2}{\partial P_3} & \dfrac{\partial f_2}{\partial D} \end{bmatrix}, \tag{22}$$

where,

$$\frac{\partial f_1}{\partial Q} = 0, \quad \frac{\partial f_2}{\partial Q} = y(1-y),$$

$$\frac{\partial f_1}{\partial Z} = x(1-x), \quad \frac{\partial f_2}{\partial Z} = 0,$$

$$\frac{\partial f_1}{\partial C_1} = x(1-x), \quad \frac{\partial f_2}{\partial C_1} = 0,$$

$$\frac{\partial f_1}{\partial P_3} = 0, \quad \frac{\partial f_2}{\partial P_3} = xy(1-y)(g(t)-h(t)),$$

$$\frac{\partial f_1}{\partial D} = \mathrm{x}(1-x)[(h(t)-g(t))y - h(t)], \quad \frac{\partial f_2}{\partial D} = 0,$$

$$\frac{\partial f_1}{\partial h(t)} = -Dx(1-x)(1-y), \quad \frac{\partial f_2}{\partial h(t)} = -P_3 xy(1-y),$$

$$\frac{\partial f_1}{\partial g(t)} = -Dxy(1-x), \quad \frac{\partial f_2}{\partial g(t)} = P_3 xy(1-y).$$

Now, let's assume the nominal values of the parameters are as follows: $Q=1$, $Z=11$, $C_1=9.2$, $h(t)=27$, $g(t)=24$, $P_3=0.25$, $D=0.8$. According to formula (9), it is obvious that the nominal system is as follows:

$$\begin{cases} \dfrac{dx}{dt} = x(1-x)(2.4y-1.4), \\ \dfrac{dy}{dt} = y(1-y)(-0.75x+1). \end{cases}$$

According to formulas (20), (21), and (22), we can obtain the matrix of partial derivatives concerning $u$, $\lambda_1$ and $\lambda_2$ at the given nominal values,

$$\left.\frac{\partial f}{\partial u}\right|_{\text{nominal}} = \begin{bmatrix} (1-2x)(2.4y-1.4) & 2.4x(1-x) \\ -0.75y(1-y) & (1-2y)(-0.75x+1) \end{bmatrix}, \tag{23}$$

$$\left.\frac{\partial f}{\partial \lambda_1}\right|_{\text{nominal}} = \begin{bmatrix} 0 & x(1-x) & x(1-x) & -0.8x(1-x)(1-y) & -0.8xy(1-x) \\ -y(1-y) & 0 & 0 & -0.25xy(1-y) & 0.25xy(1-y) \end{bmatrix}, \tag{24}$$

$$\left.\frac{\partial f}{\partial \lambda_2}\right|_{\text{nominal}} = \begin{bmatrix} 0 & 3x(1-x)(y-9) \\ -3xy(1-y) & 0 \end{bmatrix}. \tag{25}$$

The sensitivity function of the model (9) is as follows:

$$S_{\lambda_1} = \begin{bmatrix} S_1 & S_3 & S_5 & S_7 & S_9 \\ S_2 & S_4 & S_6 & S_8 & S_{10} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial Q} & \frac{\partial x}{\partial Z} & \frac{\partial x}{\partial C_1} & \frac{\partial x}{\partial h(t)} & \frac{\partial x}{\partial g(t)} \\ \frac{\partial y}{\partial Q} & \frac{\partial y}{\partial Z} & \frac{\partial y}{\partial C_1} & \frac{\partial x}{\partial h(t)} & \frac{\partial x}{\partial g(t)} \end{bmatrix}, \tag{26}$$

$$S_{\lambda_2} = \begin{bmatrix} S_{11} & S_{13} \\ S_{12} & S_{14} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial P_3} & \frac{\partial x}{\partial D} \\ \frac{\partial y}{\partial P_3} & \frac{\partial y}{\partial D} \end{bmatrix}. \tag{27}$$

According to formulas (19), (23), (24), (25), (26), and (27), we can obtain,

$$\begin{aligned}
\dot{S}_1 &= (1-2x)(2.4y-1.4)S_1 + 2.4x(1-x)S_2 \\
\dot{S}_2 &= -0.75y(1-y)S_1 + (1-2y)(-0.75x+1)S_2 - y(1-y) \\
\dot{S}_3 &= (1-2x)(2.4y-1.4)S_3 + 2.4x(1-x)S_4 + x(1-x) \\
\dot{S}_4 &= -0.75y(1-y)S_3 + (1-2y)(-0.75x+1)S_4 \\
\dot{S}_5 &= (1-2x)(2.4y-1.4)S_5 + 2.4x(1-x)S_6 + x(1-x) \\
\dot{S}_6 &= -0.75y(1-y)S_5 + (1-2y)(-0.75x+1)S_6 \\
\dot{S}_7 &= (1-2x)(2.4y-1.4)S_7 + 2.4x(1-x)S_8 - 0.8x(1-x)(1-y) \\
\dot{S}_8 &= -0.75y(1-y)S_7 + (1-2y)(-0.75x+1)S_8 - 0.25xy(1-y) \\
\dot{S}_9 &= (1-2x)(2.4y-1.4)S_9 + 2.4x(1-x)S_{10} - 0.8xy(1-x) \\
\dot{S}_{10} &= -0.75y(1-y)S_9 + (1-2y)(-0.75x+1)S_{10} + 0.25xy(1-y) \\
\dot{S}_{11} &= (1-2x)(2.4y-1.4)S_{11} + 2.4x(1-x)S_{12} \\
\dot{S}_{12} &= -0.75y(1-y)S_{11} + (1-2y)(-0.75x+1)S_{12} - 3xy(1-y) \\
\dot{S}_{13} &= (1-2x)(2.4y-1.4)S_{13} + 2.4x(1-x)S_{14} + 3x(1-x)(y-9) \\
\dot{S}_{14} &= -0.75y(1-y)S_{13} + (1-2y)(-0.75x+1)S_{14}
\end{aligned} \tag{28}$$

Assuming the initial values of the above system of differential equations are $x = 0.78$, $y = 0.3$, the results obtained through the fourth-fifth order Runge-Kutta algorithm are shown in **Figure 2**. **Figure 2**(a) represents the sensitivity of the concerning parameter $P_3, D$. **Figure 2**(b) represents the sensitivity of $y$ concerning parameter $P_3, D$. **Figure 2**(c) represents the sensitivity of $x$ with respect to parameter $Q, C_1, Z, g(t), h(t)$. **Figure 2**(d) represents the sensitivity of $y$ with respect to the parameter $Q, C_1, Z$, $g(t), h(t)$. From **Figure 2**(a) and (b), it can be seen that the sensitivity of the solution to parameter $D$ is higher compared to the sensitivities to other parameters. This implies that the discounts received by drivers when paying parking fees on the shared platform have a greater impact on the decisions of both drivers and the shared platform. Similarly, from **Figure 2**(c) and (d), it is obvious that the sensitivity of the solution to parameter $Q$ is higher compared to the sensitivities to other parameters. This indicates that the additional payoff generated by sharing for the shared platform has a more significant influence on the decisions of both drivers and the shared platform.

Based on the above analysis, sharing platforms should be extremely cautious when increasing discounts, and the government should also be extremely cautious when increasing investment in sharing platforms.

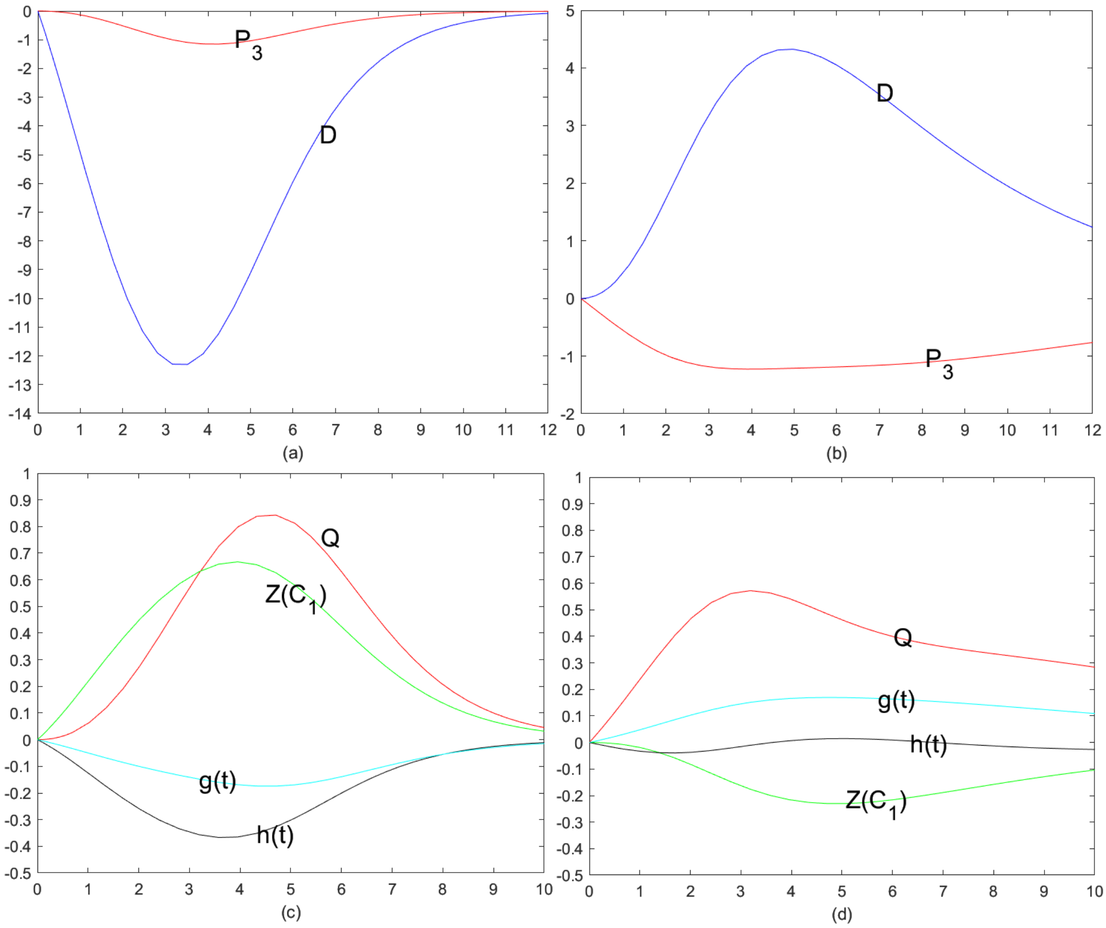



**Figure 2.** The sensitivity of the parameter $Q, Z, C_1, h(t), g(t), P_3, D$

## 5. Numerical simulation

This paper will use MATLAB software to conduct numerical simulation. The values of parameters $P_1$, $P_2$, $F$ and $U$ are determined from the reference [11]. The values of parameters $P_3$ determined from the reference [30]. According to the survey, it is reasonable to assume that the driver gets a discount of 0.8 and the additional benefit of the shared platform is 1. The average parking time of the driver is set to the normal working time of one day. **Table 5** shows the parameter values used in the simulation process.

**Table 4.** The values of the parameters for the model

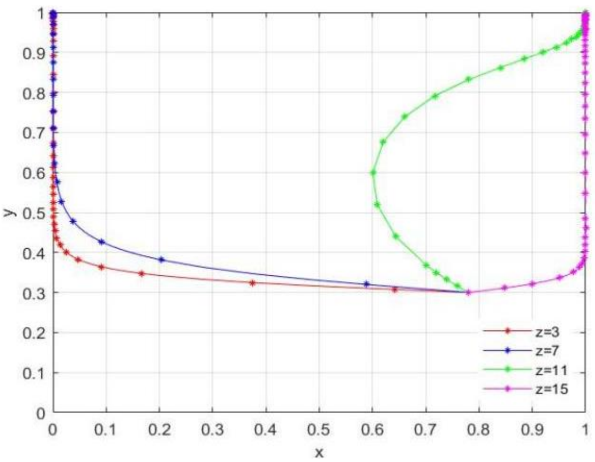
| parameter | $C_1$ | $P_1$ | $P_2$ | $F$ | $U$ | $Q$ | $P_3$ | $D$ | $\Delta t$ |
|---|---|---|---|---|---|---|---|---|---|
| value | 9.2 | 0.1 | 0.005 | 100 | 200 | 1 | 0.25 | 0.8 | 8 |

### *5.1. The influence of model parameters on the evolution path*

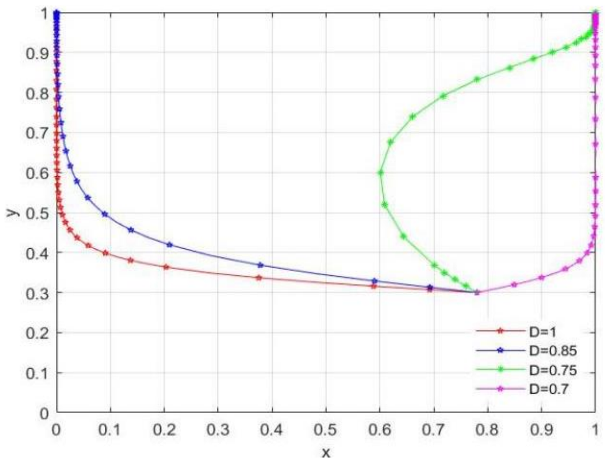
In the previous section, this paper conducts a sensitivity analysis on the model parameters and finds that the sensitivity of the discounts received by drivers when paying parking fees and the additional payoff generated by the shared platform is higher than other parameters. In this section, according to the results from the previous section, this paper will perform simulation experiments on the parameters with relatively higher sensitivity. This will allow us to explore the impact of different parameter values on the evolutionary trajectory of both players in the game.

**Figure 3** illustrates the impact of varying values of parameters $Z, P_3, Q, D$ on the evolution path while keeping other parameters fixed. From **Figure 3**(a), it can be observed that when the basic loss $Z$ for drivers' illegal parking is small, the system will evolve towards (0,1), indicating that drivers tend to choose illegal parking while the shared platform selects surrounding parking lots. This will lead to disorder in the social order. From **Figure 3**(b), it can be observed that when the discounts $D$ received by drivers for paying parking fees are small, the system will evolve towards (0,1), which also leads to disorder in the social order. From **Figure 3**(c), it can be observed that as the coefficient $P_3$ of the shared platform's revenue from parking fees increases, the number of drivers engaging in illegal parking during a certain period will increase. From **Figure 3**(d), it can be seen that when the shared platform's additional payoff $Q$ is a positive and relatively large value, the system will evolve towards the ideal ESS (1,1). However, when the additional payoff is negative or small, the system will evolve towards (0,0), indicating that drivers engage in illegal parking while the shared platform selects destination parking lots. This will severely impact the social order. By observing the four figures, we can also notice that the green line, though eventually evolving towards the ideal ESS (1,1), exhibits a tendency towards illegal parking for some drivers during the initial period.

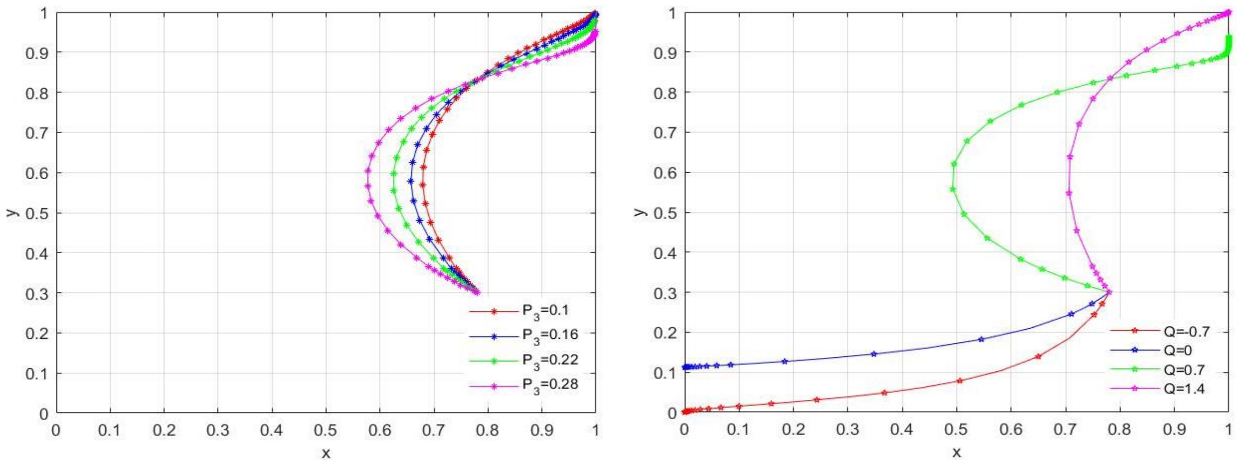
The above findings indicate that moderately increasing discounts, additional revenue from shared platforms, and fines for violations will help the model develop towards drivers choosing to park legally and shared platforms choosing surrounding parking lots, which will be the most desired outcome for society.




(a)




(b)




(c) (d)

**Figure 3.** The influence of varying values of parameters $Z, P_3, Q, D$ on the evolution path

### *5.2. Dynamic parking fee standard numerical analysis*

In this section, this paper will perform a numerical analysis of the dynamic parking fee standard $K_1$ and $K_2$. In real-world road traffic scenarios, the traffic flow exhibits peak and off-peak periods with strong periodicity. Lowering the parking fee standard during peak periods can help disperse drivers heading to the same destination for parking, thereby reducing traffic congestion. Therefore, the parking fee standard should also exhibit periodicity. To better reflect the actual traffic conditions, this paper will use trigonometric functions to simulate the dynamic parking fee standard.

Let

$$K_1 = -a\cos(\frac{\pi}{6}t - \frac{\pi}{3}) + b,$$

$$K_2 = -c\cos(\frac{\pi}{6}t - \frac{\pi}{3}) + d,$$

where $b$ and $d$ represent the basic parking fee standard, $a$ and $c$ represent the maximum difference between the actual parking fee standard and the basic parking fee standard. **Figure 4** shows the variation of parking fees in destination parking lots over time.

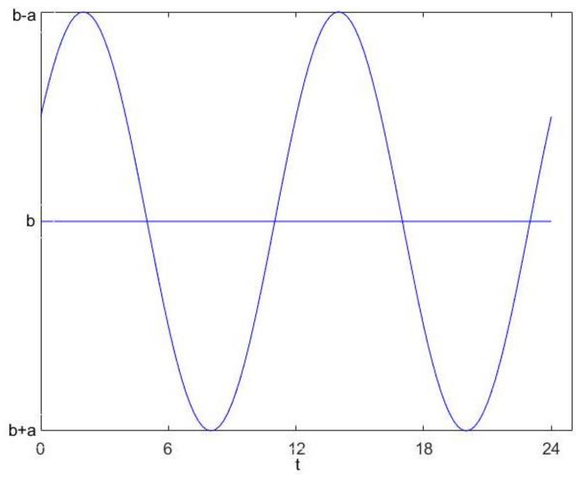



**Figure 4.** Destination parking fee standard change over time( $a < 0$ )

#### 5.2.1. The influence of parameters of dynamic parking fee standard on evolutionary paths

**Figure 5** and **Figure 6** illustrate the stable evolution directions of drivers and the shared platform under different parameter variations.

**Figure 5** shows the impact of changing the basic fee standard $b$ for the destination parking lot and $d$ for surrounding parking lots on the evolutionary dynamics of $x$ and $y$. In this figure, **Figure 5**(a) and (b) show the evolution paths of the driver's probability of parking legally and the shared platform's probability of choosing surrounding parking lots under varying $b$ while keeping $a = 0.1, c = 0.1, d = 3.0$ constant. In addition, **Figure 5**(c) and (d) show the evolution paths of the driver's probability of parking legally and the shared platform's probability of choosing surrounding parking lots under varying $d$ while keeping $a = 0.1, c = 0.1, b = 3.4$ constant. From **Figure 5**(a), it can be observed that as the basic parking fee standard $b$ for destination parking lots decreases, the convergence rate of the driver's probability of parking legally towards 1 increases, and when $b$ falls over a certain threshold, drivers tend to engage in illegal parking during the initial period. From **Figure 5**(b), it can be observed that as the value of $b$ decreases, the convergence rate of the shared platform's probability of choosing surrounding parking lots towards 1 increases. This indicates that the lower the basic parking fee standard for surrounding parking lots, the faster it evolves towards a favorable direction, which is drivers parking legally and the shared platform choosing surrounding parking lots. From **Figure 5**(c), it can be observed that as the value of $d$ increases, the convergence rate of the driver's probability of parking legally towards 1 slows down, and when $d$ is smaller than a certain threshold, drivers tend to engage in illegal parking. From **Figure 5**(d), it can be observed that as the value of $d$ increases, the convergence rate of the shared platform's probability of choosing surrounding parking lots towards 1 becomes faster. This indicates that there exists a value of $d$ where both the driver's probability of parking legally and the shared platform's probability of choosing surrounding parking lots converge to 1 at a relatively fast rate.

**Figure 6** shows the impact of changing the maximum difference between the actual parking fee and the basic parking fee standard $a$ for the destination parking lot and $c$ for surrounding parking lots on the evolutionary dynamics of $x$ and $y$. In this figure, **Figure 6**(a) and (b) show the evolution paths of the driver's probability of parking legally and the shared platform's probability of choosing surrounding parking lots under varying $a$ while keeping $b = 3.4, c = 0.1, d = 3.0$ constant. In addition, **Figure 6**(c) and (d) show the evolution paths of the driver's probability of parking legally and the shared platform's probability of choosing surrounding parking lots under varying $c$ while keeping $a = 0.1, b = 3.4, d = 3.0$ constant. From **Figure 6**(a), it is evident that as the maximum difference $a$ increases, the convergence rate of the driver's probability of parking legally towards 1 decreases. It is also noticeable that initially, some drivers tend to engage in illegal parking, but after some time of learning, they will tend to park legally. These findings indicate that a smaller value of $a$ favors the drivers to park legally. **Figure 6**(b) shows a change in the value of $a$ has little effect on the decision of the shared platform. From **Figure 6**(c), it is evident that a smaller value of $c$ results in a faster convergence of the driver's probability of parking legally towards 1. However, when $c$ is smaller than a certain threshold, the driver's probability of parking legally will not converge to 1. From **Figure 6**(d), it is evident that as $c$ decreases, the convergence rate of the shared platform's probability of choosing surrounding parking lots towards 1 slows down. These findings indicate that there exists a value of $c$ where both the driver's probability of parking legally and the shared platform's probability of choosing surrounding parking lots converge to 1 at a relatively fast rate.

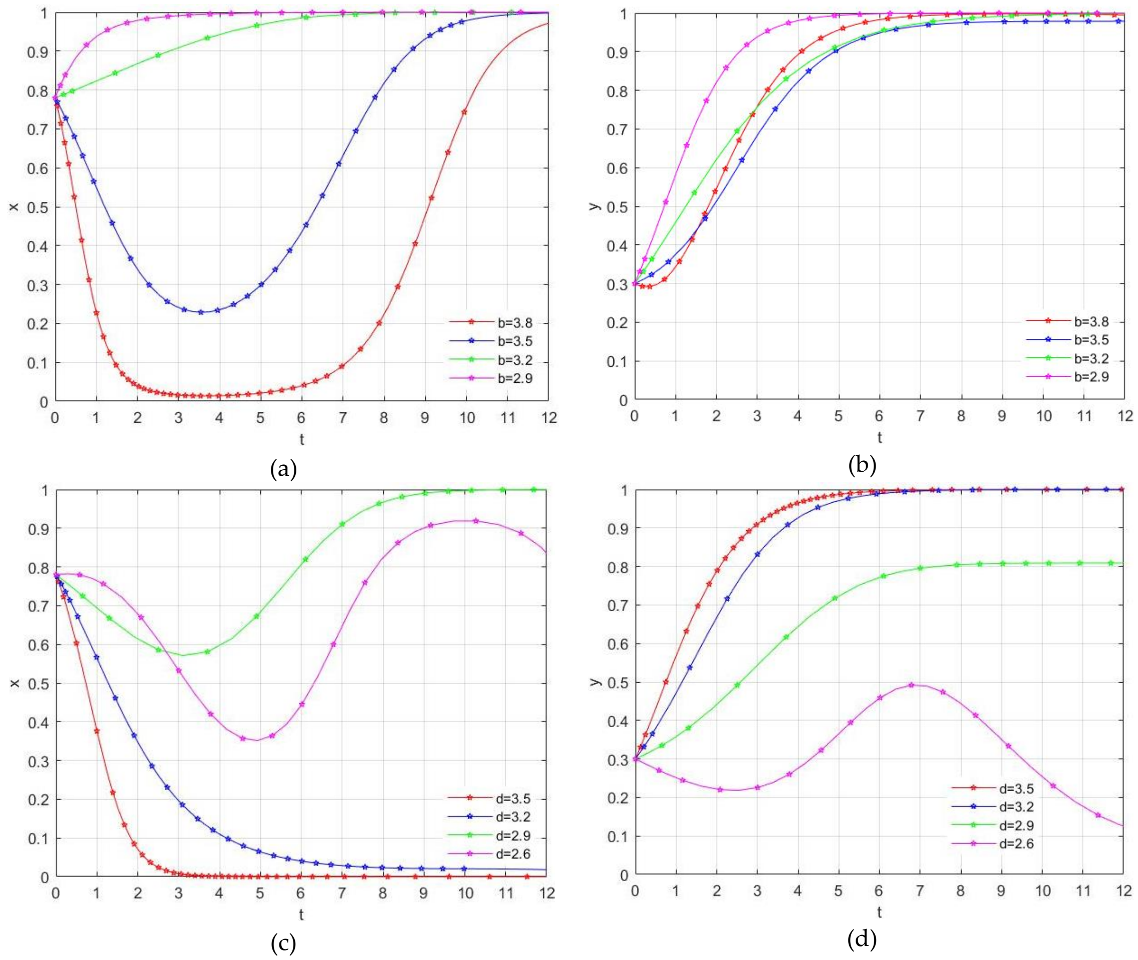
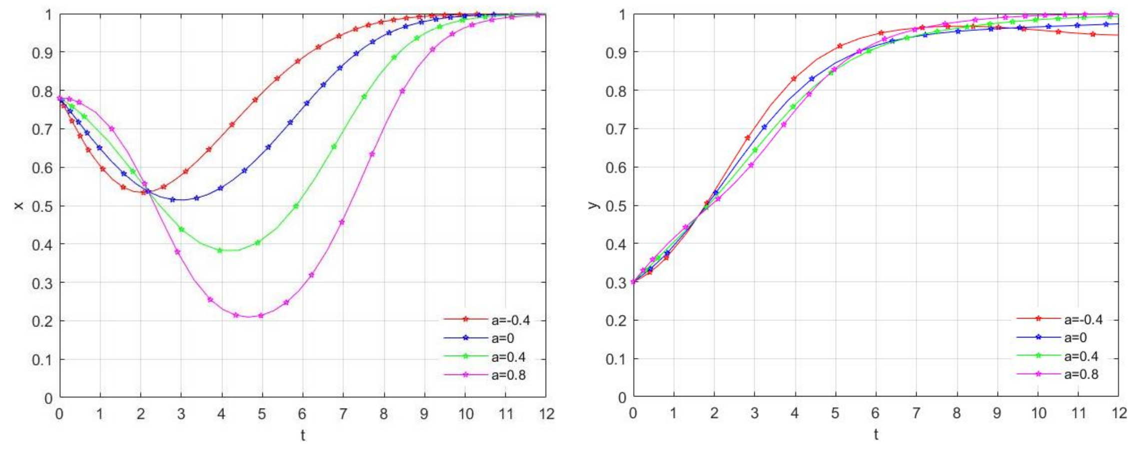
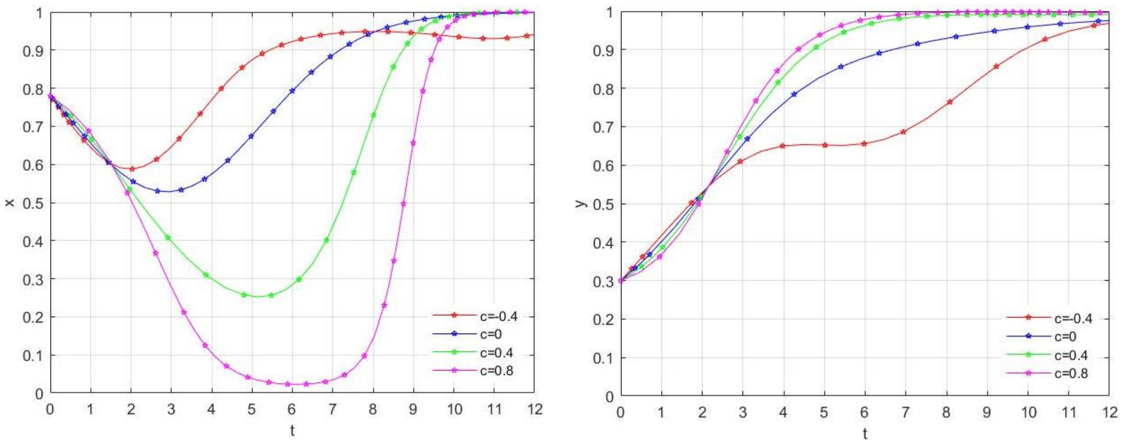



**Figure 5.** The impact of the basic parking fee on the evolution dynamics of $x$ and $y$






(c) (d)

**Figure 6.** The impact of the maximum difference between the actual parking fee and the basic parking fee on the evolution dynamics of $x$ and $y$

#### 5.2.2. Local optimal dynamic parking fee standard based on particle swarm optimization algorithm

Particle swarm optimization was first proposed by Kennedy and Eberhardt in 1995 as a population-based optimization algorithm [30]. It has good global search ability, fewer parameter settings, and simple implementation, therefore, it is widely used in other fields such as function optimization and machine learning [31].

To enable the model to evolve at a faster rate to drivers choosing legal parking and the shared platform choosing surrounding parking lots, this paper needs to find a set of $a,b,c$ and $d$, so that the probability of drivers' legal parking and the probability of shared platform choosing surrounding parking lots converge to 1 at a faster rate. In other words, this paper aims to maximize the probability of drivers choosing to park legally and the probability of shared platforms choosing surrounding parking lots at each moment. For the convenience of solving, this paper uses the sum of the driver's legal parking probability at all times from the start of the game to $T$ time and the sum of the probability of the shared platform selecting surrounding parking lots at all times from the start of the game to $T$ time as the objective function.

To obtain the objective function, this paper needs to discretize the shared parking model (9). There, this paper will use the backward difference method and take the step size h to obtain the following result,

$$\begin{aligned} x(t+h) &= x(t) + hx(t)(1-x(t))[D(h(t)-g(t))y(t) - Dh(t) + C_1 + Z], \\ y(t+h) &= y(t) + hy(t)(1-y(t))[P_3(g(t)-h(t))x(t) + Q]. \end{aligned} \tag{29}$$

According to formula (29), gradually iterating and summing, this paper can finally obtain the sum of the driver's legal parking probability and the sum of the shared platform's probability of selecting surrounding parking lots, which are $G(a,b,c,d)$ and $H(a,b,c,d)$, respectively.

$$G(a,b,c,d) = \sum_{i=0}^{n} x(ih), \quad H(a,b,c,d) = \sum_{i=1}^{n} y(ih),$$

where, $n = \dfrac{T}{h}$.

By assigning weights respectively, we obtain the objective function as follows,

$$W(a,b,c,d) = 0.6G + 0.4H. \tag{30}$$

Next, we will solve the optimal solution of formula (30) based on the PSO algorithm.

Firstly, we need to set the parameters of the PSO algorithm. The number of independent variables in the objective optimization function is $D_1$, the number of particles is $M$, the position and velocity of the ith particle are $L_i$ and $V_i$, respectively. The ith particle searches for the best position every time as $L_{ibest}$, and the entire population searches for the best position as $L_{gbest}$. Secondly, for particles to have the ability to approach the optimal solution, we need to update their velocity. The weight of the particle inheriting the original speed is $w$, and the individual learning factor and group learning factor are $b_1$ and $b_2$, respectively. Meanwhile, due to the characteristics of the algorithm, in order to avoid the algorithm falling into local optima, we introduce random numbers $r_1$ and $r_2$. Finally, we provide the speed update formula as follows,

$$v_i^{j+1} = wv_i^j + b_1 r_1 (L_{ibest}^j - L_i^j) + b_2 r_2 (L_{gbest}^j - L_i^j).$$

Meanwhile, in order to balance the exploration and development capabilities of the algorithm, the maximum and minimum velocities of particles are limited to $v_{\max}$ and $v_{\min}$, respectively. If the updated particle velocity is not within the limit range, update the velocity according to the following formula,

$$v_i^{j+1} = r(v_{\max} - v_{\min}) + v_{\min},$$

where, $r$ is a random number and $r \in [0,1]$.

After updating the particle velocity, it is necessary to update the position of the particles, as shown in the formula (31). To ensure that the particle position does not exceed the range of the solution, the particle position is limited between $L_{\min}$ and $L_{\max}$. If the position of the particle exceeds the range after updating, update the particle position according to formula (32),

$$L_i^{k+1} = L_i^k + v_i^{k+1}, \tag{31}$$

$$L_i^k = r(L_{\max} - L_{\min}) + L_{\min}. \tag{32}$$

After updating the particle speed and position, according to formula (30), we can calculate the fitness value (objective function value) for each particle's location. If the objective function value of the particle obtained after the k-th+1st position update is higher than the kth, then the optimal position searched for by the particle is the position after the k-th+1st update. To make the algorithm be terminated and the accuracy of the solution is high, this paper sets the maximum number of iterations to $N$ (each particle searches for the optimal solution $N$ times).

**Figure 7** shows the flow of the PSO algorithm. **Table 5** shows the parameter values required for the PSO algorithm.

Finally, we have obtained that under the conditions of $a = -0.363$, $b = 2.8$, $c = 0.193$ and $d = 3.081$, both the driver's probability of parking legally and the shared platform's probability of choosing surrounding parking lots converge to 1 at a relatively fast rate, as shown in **Figure 8**.

**Table 5.** The parameter values required for the algorithm

| parameter | $D_1$ | $M$ | $N$ | $w$ | $b_{1,2}$ | $T$ | $v_{\min}$ | $v_{\max}$ | $L_{\min}$ | $L_{\max}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| value | 4 | 50 | 100 | 0.8 | 0.8 | 8 | 0.01 | 0.03 | (-0.4,2.8,-0.4,2.8) | (0.4,3.2,0.4,3.2) |

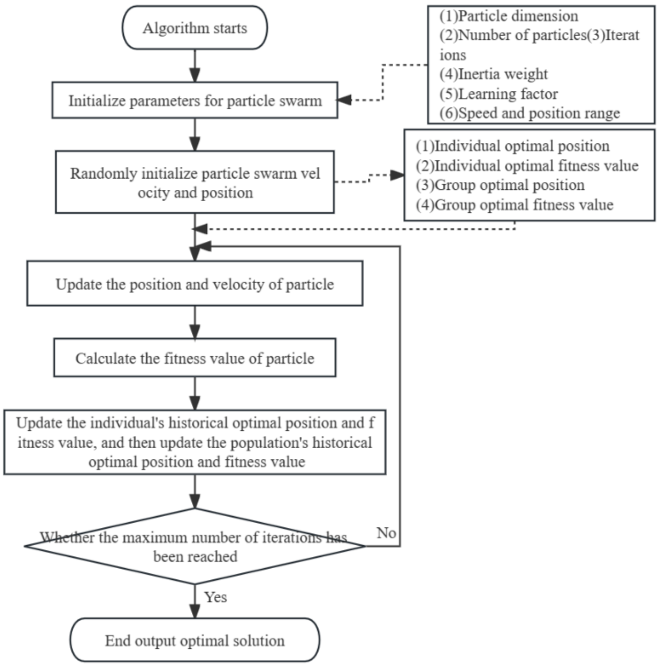



**Figure 7.** Flow chart of particle swarm optimization algorithm

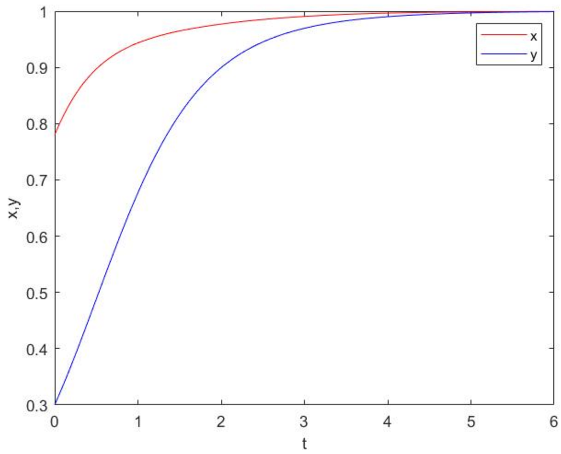



**Figure 8.** The evolutionary path of $x$ and $y$ under the local optimal parking fee standard

## 6. Conclusion

This paper establishes an evolutionary game model between the sharing platform and motor vehicle drivers, investigates the evolutionarily stable strategies of the model, conducts sensitivity analysis on the parameters, and explores the impact of highly sensitive parameters on the decisions of drivers and the sharing platform. Finally, we simulate the parking fee schedule with unknown parameters using trigonometric functions and explore the influence of different parameters on the evolutionary paths of drivers and the sharing platform. The main conclusions are as follows:

(1) After categorizing the parameters into cost-type and proportion-type, by sensitivity analysis, we revealed that the sensitivity of the discounts obtained by drivers when paying parking fees and the additional revenue gained by the sharing platform through sharing is higher than other parameters in the same category. This indicates that adjusting the discounts for drivers and the additional revenue for the sharing platform can lead the system to evolve more rapidly towards a stable state where drivers choose legal parking

and the sharing platform selects surrounding parking lots. However, it also reflects that excessive adjustments to these two parameters can easily disrupt social order.

(2) Through numerical simulations, it is discovered that moderately increasing the intensity of discounts for drivers, the additional payoff for the sharing platform, and penalty fines for violations will facilitate the system's evolution towards a stable state where drivers choose legal parking and the sharing platform selects surrounding parking lots. This will help solve the problem of difficult parking and promote the development of shared parking. However, if the additional income for the sharing platform is less than its expenses, it could lead to severe disruption of social order and significant economic losses.

(3)After using a trigonometric function with unknown parameters to simulate parking fare standard, based on the PSO algorithm, this paper ultimately discovers that the model evolves rapidly towards a stable state where drivers adopt legal parking and the sharing platform chooses surrounding parking lots when the parking fare standard are set as $K_1 = -0.363\cos(\frac{\pi}{6}t - \frac{\pi}{3}) + 2.8$ and $K_2 = 0.193\cos(\frac{\pi}{6}t - \frac{\pi}{3}) + 3.081$. This would be more conducive to maintaining social order and stability and provide a theoretical reference for the government to formulate the parking charge standard.

Based on our findings, we can offer the following recommendations:

(1) The government should fully leverage its guiding and incentivizing role, providing strong support to sharing platforms. Through various media channels, it should conduct extensive promotions to highlight its advantages. Market marketing and promotional activities should be utilized to attract more users to use these sharing platforms.

(2) From a green and sustainable perspective, the shared parking industry can bring significant economic and environmental benefits. Companies should clearly understand the advantages and shortcomings of shared parking, seeking advantages and avoiding disadvantages. By expanding their business in the shared parking industry, companies should also take on social responsibilities and increase their potential gains.

(3) The government can establish temporary parking lots in areas where they are needed to offer citizens short-term parking options. These parking lots can be equipped with lower fare standard to meet the demands of short-term parking needs.

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